In a triangle ABC if A,B,C are in A.P with sin(2A+B)=sin(C-A)=sin(B+2C)=1/2 then what are the values of A,B and C?

Solution:

If A, B, C are in A.P then

$$A = a,$$

$$B = a + b,$$

$$C = a + 2b.$$

Suppose that $A \leq B \leq C$. Then $b \geq 0$. Also we have

$$A + B + C = \pi,$$

$$a + a + b + a + 2b = \pi,$$

$$3a + 3b = \pi,$$

$$a + b = \frac{\pi}{3}.$$

So

$$\boxed{B = a + b = \frac{\pi}{3}}$$

Further

$$\sin(2\mathrm{A} + \mathrm{B}) = \frac{1}{2}.$$

Because $A > 0$ and $B = \frac{\pi}{3}$ then

$$2A + \frac{\pi}{3} = \pi - \frac{\pi}{6},$$

$$\boxed{A = \frac{\pi}{4}}$$

So

$$\begin{cases} a = \dfrac{\pi}{4}, \\ a + b = \dfrac{\pi}{3}. \end{cases}$$

Thus we have $b = \frac{\pi}{3} - \frac{\pi}{4} = \frac{\pi}{12}$ and

$$C = a + 2b = \frac{\pi}{4} + \frac{\pi}{6} = \frac{5\pi}{12}.$$

By the condition of the task we have

$$\sin(B + 2C) = \frac{1}{2}.$$

But there is

$$\sin(B + 2C) = \sin\left(\frac{\pi}{3} + \frac{5\pi}{6}\right) = -\frac{1}{2}.$$

Thus values of A, B, C do not exist.

Answer: Values of A, B, C do not exist.