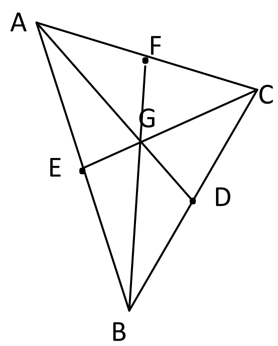
Question 1.

Find the coordinates of the centroid of the triangle having vertices at (-1,5), (1,-2) and (3,3).

Solution:



A(-1, 5), B(1, -2), C(3, 3).

Coordinates (x, y) of the middle of the line segment with endpoints $A_1(x_1, y_1)$ and $A_2(x_2,y_2)$ are

$$x = \frac{x_1 + x_2}{2}, \qquad y = \frac{y_1 + y_2}{2}.$$

Let's find coordinates of the middle of the line segments AB, AC, BC.

$$x_E = \frac{x_A + x_B}{2} = \frac{-1 + 1}{2} = 0, \qquad y_E = \frac{y_A + y_B}{2} = \frac{5 - 2}{2} = \frac{3}{2}.$$

$$E(x_E, y_E) = E\left(0, \frac{3}{2}\right).$$

$$x_F = \frac{x_A + x_C}{2} = \frac{-1 + 3}{2} = 1, \qquad y_F = \frac{y_A + y_C}{2} = \frac{5 + 3}{2} = 4.$$

$$F(x_F, y_F) = F(1, 4).$$

$$x_D = \frac{x_C + x_B}{2} = \frac{3 + 1}{2} = 2, \qquad y_D = \frac{y_C + y_B}{2} = \frac{3 - 2}{2} = \frac{1}{2}.$$

$$D(x_D, y_D) = D\left(2, \frac{1}{2}\right).$$

AD, CE, BF are the medians.

Let's find the equation for lines AD, CE, BF:

The equation for non-vertical lines is often given in the slope-intercept form:

$$y = mx + c$$

Where:

m is the slope of the line.

c is the y-intercept of the line.

x is the independent variable of the function y = f(x).

Therefore,

AD:

$$y = mx + c$$

$$D\left(2, \frac{1}{2}\right), \qquad A(-1, 5)$$

$$\begin{cases} \frac{1}{2} = 2m + c \\ 5 = -m + c \end{cases} \qquad \begin{cases} 1 = 4m + 2c \\ 10 = -2m + 2c \end{cases} => m = -\frac{3}{2}, c = \frac{7}{2}$$

$$y_{AD} = -\frac{3}{2}x + \frac{7}{2}.$$

CE:

$$y = mx + c$$

C(3, 3), $\quad E\left(0, \frac{3}{2}\right)$

$$\begin{cases} \frac{3}{2} = c \\ 3 = 3m + c \end{cases} \qquad \begin{cases} \frac{3}{2} = c \\ m = \frac{1}{2} \end{cases}$$

$$y_{CE} = \frac{1}{2}x + \frac{3}{2}.$$

BF:

$$y = mx + c$$

B(1, -2), $\qquad F(1, 4).$

$$x_b = 1 \; and \; x_F = 1 => BF \; is \; a \; vertical \; line \; x = 1.$$

The three medians intersect in a single point, the triangle’s centroid.

$$\begin{cases} y_{AD} = -\frac{3}{2}x + \frac{7}{2} \\ y_{CE} = \frac{1}{2}x + \frac{3}{2} \\ x = 1 \end{cases} \qquad \begin{cases} y_{AD} = 2 \\ y_{CE} = 2 \\ x = 1 \end{cases}$$

G is a centroid of the triangle. So, G(1, 2).