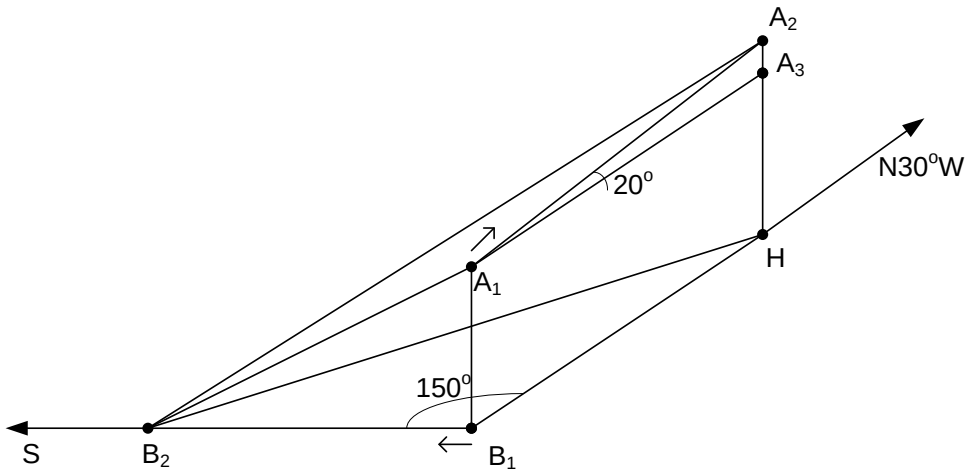
An airplane headed 30 degrees northeast climbing on a 20 degrees incline at a speed of 240 miles per hour passes 1 mile directly over a car headed south on a flat road at the rate of 60 miles per hour at what rate of speed are they separating after 10 minutes?

Solution:



B_1 - the point at which a car was when an airplane was flying over it

A_1 - the point at which there was an airplane as it flew over the car

B_2 - the point at which a car was after t hours

A_2 - the point at which an airplane was after t hours

$S = A_2B_2$ - the distance between an airplane and a car after t hours

$A_1B_1 = 1\ ml$

$B_1B_2 = 60t$

$A_1A_2 = 240t$

$HA_2 = 1 + A_1A_2 \sin 20° = 1 + 240 \sin 20° t = 1 + 82t$

$HB_2{}^2 = B_1B_2{}^2 + B_1H^2 - 2 \cdot B_1B_2 \cdot B_1H \cos 150°$

$B_1H = A_1A_3 = A_1A_2 \cos 20° = 240 \cos 20° t = 226t$

$HB_2{}^2 = (60t)^2 + (226t)^2 - 2 \cdot 60t \cdot 226t \cos 150° = 78{,}163t^2$

$S^2 = A_2B_2{}^2 = HB_2{}^2 + HA_2{}^2 = 78{,}163t^2 + (1 + 82t)^2 = 84{,}887t^2 + 164t + 1$

$S(t) = \sqrt{84{,}887t^2 + 164t + 1}$

The rate of speed is:

$$v(t) = \frac{dS}{dt} = \frac{169{,}774t + 164}{2\sqrt{84{,}887t^2 + 164t + 1}}$$

When $t = 10\ min = \frac{1}{6}\ hour$

$$v\left(\frac{1}{6}\right) = \frac{\frac{169{,}774}{6} + 164}{2\sqrt{\frac{84{,}887}{36} + \frac{164}{6} + 1}} = 291\ ml/hour$$