

The half-angle formula for the sine function:

$$\sin \frac{\alpha}{2} = \pm \frac{\sqrt{1 - \cos \alpha}}{2}$$

$$\sin \frac{5x}{8} \Rightarrow \alpha = \frac{5x}{4} ,$$

Applying the half-angle formula we have:

$$\sin \frac{5x}{8} = \pm \frac{\sqrt{1 - \cos \frac{5x}{4}}}{2}$$

The sign of the right part of the equation will be:

- positive if $0 \leq \frac{\alpha}{2} \leq \pi$ and hence if $0 \leq x \leq \frac{4\pi}{5}$;
- negative if $\pi \leq \frac{\alpha}{2} \leq 2\pi$, so $\frac{4\pi}{5} \leq x \leq \frac{8\pi}{5}$.