Question: Kara and Allan can complete a job in 2 hours. Allan and Becky can do the job in 3 hours. Kara and Becky can do the same job in 4 hours. How many hours will it take all three working together to complete the job?

Solution:

Let the speed with which Kara does the job (J) alone is x_k, the speed with which Allan does the job (J) alone is x_a and the speed with which Becky does the job (J) alone is x_b, then

$$\begin{cases} 2(x_k + x_a) = J \\ 3(x_a + x_b) = J \\ 4(x_k + x_b) = J \end{cases} (1)$$

In question need to find how many hours will it take all three working together to complete the job:

$$t_{a,b,k} = \frac{J}{x_a + x_b + x_k}$$

Using (1) express x_a, x_b, x_k via J:

$$\begin{cases} x_a = \frac{J}{2} - x_k \\ x_a + x_b = \frac{J}{3} \\ x_b = \frac{J}{4} - x_k \end{cases}$$

Substitute first and third equation into second:

$$\frac{J}{2} - x_k + \frac{J}{4} - x_k = \frac{J}{3}$$

$$2x_k = \frac{5J}{12}$$

$$x_k = \frac{5J}{24}$$

$$x_a + x_b + x_k = \frac{J}{3} + \frac{5J}{24} = \frac{13J}{24}$$

So,

$$t_{a,b,k} = \frac{J}{x_a + x_b + x_k} = \frac{J}{13J/24} = \frac{24}{13} \approx 1.85 \text{ hours}$$

Answer: 1.85 hours