

if $X\sqrt{1+Y}+Y\sqrt{1+X}=0$ then find dy/dx

$$X\sqrt{1+Y}+Y\sqrt{1+X}=0 \Rightarrow x\sqrt{1+y} + y\sqrt{1+x} = 0$$

$$\text{find } d(x\sqrt{1+y} + y\sqrt{1+x}) = d(0)$$

0 is constant, so $d(0) = 0$

$$d(x\sqrt{1+y} + y\sqrt{1+x}) = 0$$

The sum rule:

$$d(x\sqrt{1+y}) + d(y\sqrt{1+x}) = 0$$

The product rule (Leibniz rule):

$$\sqrt{1+y}dx + xd(\sqrt{1+y}) + \sqrt{1+x}dy + yd(\sqrt{1+x}) = 0$$

$$d(\sqrt{1+y}) = \frac{dy}{2\sqrt{1+y}}$$

Finally:

$$\sqrt{1+y}dx + x\frac{dy}{2\sqrt{1+y}} + \sqrt{1+x}dy + y\frac{dx}{2\sqrt{1+x}} = 0$$

Or:

$$dx\left(\sqrt{1+y} + \frac{y}{2\sqrt{1+x}}\right) = -dy\left(\sqrt{1+x} + \frac{x}{2\sqrt{1+y}}\right)$$

$$\frac{dy}{dx} = -\frac{\left(\sqrt{1+y} + \frac{y}{2\sqrt{1+x}}\right)}{\left(\sqrt{1+x} + \frac{x}{2\sqrt{1+y}}\right)}$$