Question #27620

The midpoints of a triangle is (1,5,-1) (0,4,-2) and (2,3,4) find its vertices

Solution. Let ABC be a triangle and $M(1,5,-1), N(0,4,-2)$ and $K(2,3,4)$ be the midpoints of the sides AB, BC and AC, respectively. Denote the coordinates of the vertices by $A(x_1, y_1, z_1), B(x_2, y_2, z_2), C(x_3, y_3, z_3)$. Then, using the formula for the coordinates of a midpoint, we obtain:

$$\begin{cases} \dfrac{x_1 + x_2}{2} = 1 \\ \dfrac{x_1 + x_3}{2} = 2 \\ \dfrac{x_2 + x_3}{2} = 0 \end{cases}$$

It follows that $x_2 = -x_3$ and $x_1 = 4 - x_3$. The first equation implies that $4 - 2x_3 = 2$ and so $x_3 = 1$.

Then $x_2 = -1, x_1 = 3$.

By analogy we have

$$\begin{cases} \dfrac{y_1 + y_2}{2} = 5 \\ \dfrac{y_1 + y_3}{2} = 3 \\ \dfrac{y_2 + y_3}{2} = 4 \end{cases}$$

Then $y_2 = 10 - y_1, y_3 = 6 - y_1$. It follows from the third equation that $y_1 = 4$ and so $y_2 = 6, y_3 = 2$.

By the same method, we obtain

$$\begin{cases} \dfrac{z_1 + z_2}{2} = -1 \\ \dfrac{z_1 + z_3}{2} = 4 \\ \dfrac{z_2 + z_3}{2} = -2 \end{cases}$$

Thus $z_2 = -2 - z_1$ and $z_3 = 8 - z_1$. The third equation implies that $2z_1 = 10, z_1 = 5$. Thus $z_2 = -7, z_3 = 3$.

Finally, $A(3,4,5), B(-1,6,-7), C(1,2,3)$.

Answer. $(3,4,5), (-1,6,-7), (1,2,3)$.