Task. Prove

$$\frac{\sin(\theta)}{1-\cos(\theta)} = \csc(\theta) + \cot(\theta).$$

Proof. Notice that

$$\csc(\theta) + \cot(\theta) = \frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta},$$

and so the identity which we need to prove implies that $\sin\theta \neq 0$. Therefore $\cos\theta \neq \pm 1$. Hence $1+\cos\theta \neq 0$, and we can multiply the numerator and denominator of the fraction

$$\frac{\sin(\theta)}{1-\cos(\theta)}$$

by $1+\cos\theta$:

$$\begin{aligned}\frac{\sin\theta}{1-\cos\theta} &= \frac{\sin\theta(1+\cos\theta)}{(1-\cos\theta)(1+\cos\theta)} = \frac{\sin\theta(1+\cos\theta)}{1-\cos^2\theta} \\ &= \frac{\sin\theta(1+\cos\theta)}{\sin^2\theta} = \frac{1+\cos\theta}{\sin\theta} \\ &= \frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta} = \csc(\theta) + \cot(\theta).\end{aligned}$$