Let A={1,1/2,1/4,1/8,…} and B={ 1/2,3/4,7/8,…}. Explain why supA=supB=1?

Solution.

By the definition $\sup A$ is the least number s such that $a_n \le s$ for any $a_n \in A$. Obviously, value $s=1$ satisfies this condition.

Consider now the sequence $B=\{b_n\}$, here $b_n=(2^n-1)/2^n, n=1,2,\ldots$ and suppose that $\sup B = s<1$. Note that b_n increases, $b_n<1$ and $\lim_{n\to\infty} b_n = 1$. It means that any ε-vicinity of 1 contains elements of this sequence. Let's take ,for example, $\varepsilon = 1-s$ and suppose that the terms b_n belong to this vicinity. But then it follows that $b_n > 1-\varepsilon = 1-(1-s)=s$ and by this reason $\sup B$ can't be less than 1.

Thus, $\sup A = \sup B = 1$.