

Question #27434

$$2\sin x/\sin 3x + \tan x/\tan 3x = 1$$

we all know that

$$\sin 3x = 3 \sin x - 4 \sin^3 x \quad [1]$$

$$\cos 3x = 4 \cos^3 x - 3 \cos x \quad [2]$$

$$\tan x = \sin x / \cos x \quad [3]$$

$$\sin^2 x + \cos^2 x = 1 \quad [4]$$

substituting this into the problem statement

$$\begin{aligned} \frac{2\sin x}{3\sin x - 4\sin^3 x} + \frac{\frac{\sin x}{\cos x}}{\frac{3\sin x - 4\sin^3 x}{4\cos^3 x - 3\cos x}} &= \frac{2\sin x}{3\sin x - 4\sin^3 x} + \frac{\sin x(4\cos^3 x - 3\cos x)}{\cos x(3\sin x - 4\sin^3 x)} = \\ &= \frac{2\sin x * \cos x + 4\sin x * \cos^3 x - 3\cos x * \sin x}{(3\sin x - 4\sin^3 x) * \cos x} = \\ &= \frac{\cos x(2\sin x + 4\sin x * \cos^2 x - 3\sin x)}{(3\sin x - 4\sin^3 x) * \cos x} = \frac{\sin x(4\cos^2 x - 1)}{\sin x(3 - 4\sin^2 x)} \end{aligned}$$

Using formula [4]

$$4 \cos^2 x = 4 - 4\sin^2 x$$

And substituting the result we get

$$\frac{4 - 4\sin^2 x - 1}{3 - 4\sin^2 x} = \frac{3 - 4\sin^2 x}{3 - 4\sin^2 x} = 1$$