Write the matrix $A = [3,-1,1,-2]$ as a linear combination of $A1 = [1,1,0,-1]$, $A2 = [1,1,-1,0]$and $A3 = [1,-1,0,0]$

Solution:

We need to present the matrix A in the form:

$$A = x * A_1 + \mathrm{y} * A_2 + \mathrm{z} * A_3$$

where x, y, z some constants.

$$\begin{aligned} x * A_1 + \mathrm{y} * A_2 + \mathrm{z} * A_3 &= \mathrm{x} * [1,1,0,-1] + \mathrm{y} * [1,1,-1,0] + z * [1,-1,0,0] \\ &= [x, x, 0, -x] + [y, y, -y, 0] + [z, -z, 0,0] \\ &= [x + y + z, x + y - z, 0 - y + 0, -x + 0 + 0] \\ &= [x + y + z, x + y - z, -y, -x] \end{aligned}$$

So we have that

$$[x + y + z, x + y - z, -y, -x] = \mathrm{A}$$

$$[x + y + z, x + y - z, -y, -x] = [3,-1,1,-2]$$

We have next system of linear equation:

$$\begin{cases} x + y + z = 3 \\ x + y - z = -1 \\ -y = 1 \\ -x = -2 \end{cases}$$

$$\begin{cases} 2z = 4 \\ y = -1 \\ x = 2 \end{cases}$$

$$\begin{cases} x = 2 \\ y = -1 \\ z = 2 \end{cases}$$

So matrix A can be present as

$$A = 2A_1 - A_2 + 2A_3$$

Answer: $A = 2A_1 - A_2 + 2A_3$