

Hello Expert. Sir please solve the following question. Suppose we have a quadratic equation

$$x^2 - bx + 2 = 0$$

whose roots are consecutive integers. Then what is discriminant in this case. I tried to solve it and supposed two consecutive roots are n and $n+1$. Then sum of roots $=2n+1$ and product of roots $=n^2+n$. Then I formed a quadratic equation whose discriminant is 1 but not sure. I will be patient for your reply. Best regards.

Solution:

We have next equation

$$x^2 - bx + 2 = 0.$$

Suppose x_1, x_2 are roots of the equation. Then

$$\begin{cases} x_1 + x_2 = b, \\ x_1 \cdot x_2 = 2. \end{cases}$$

If x_1 and x_2 are consecutive **integers** then

$$x_1 \cdot x_2 = 2$$

if and only if

$$x_1 = 1, x_2 = 2 \text{ (} x_1 = 2, x_2 = 1 \text{)}$$

or

$$x_1 = -1, x_2 = -2 \text{ (} x_1 = -2, x_2 = -1 \text{)}$$

Thus we have

$$b = x_1 + x_2 = 1 + 2 = 3$$

or

$$b = x_1 + x_2 = -1 - 2 = -3.$$

Discriminant is

$$D = b^2 - 4ac = (\pm 3)^2 - 4 \cdot 2 = 9 - 8 = 1.$$