Solve $sin3x + cos2x = -2$

Solution:

As we know $sin\alpha \geq -1$ for any α, also $cos\beta \geq -1$ for any β.

So the maximum value of the expression $sin\alpha + cos\beta \geq -1 - 1$ is -2, and it is only if $sin\alpha = -1\ and\ cos\beta = -1$

So we have next system:

$$\begin{cases} sin3x = -1 \\ cos2x = -1 \end{cases}$$

$$\begin{cases} 3x = -\frac{\pi}{2} + 2\pi n, n \in Z \\ 2x = \pi + 2\pi k, k \in Z \end{cases}$$

$$\begin{cases} x = -\frac{\pi}{6} + \frac{2\pi n}{3}, n \in Z \\ x = \frac{\pi}{2} + \pi k, k \in Z \end{cases}$$

$$\begin{cases} \frac{\pi}{2} + \pi k = -\frac{\pi}{6} + \frac{2\pi n}{3}, n, k \in Z \\ x = \frac{\pi}{2} + \pi k, k \in Z \end{cases}$$

We must solve the first equation in integers

$$\begin{cases} k = -\frac{1}{6} + \frac{2n}{3} - \frac{1}{2}, n, k \in Z \\ x = \frac{\pi}{4} + \pi k, k \in Z \end{cases}$$

$$\begin{cases} k = \frac{2n - 2}{3}, n, k \in Z \\ x = \frac{\pi}{2} + \pi k, k \in Z \end{cases}$$

$$\begin{cases} 3k = 2n - 2, n, k \in Z \\ x = \frac{\pi}{2} + \pi k, k \in Z \end{cases}$$

The right sight of equation 1 is even, so $3k$ must be even too, so k must be even.
If k is even it can be present as $k = 2 * p$ where $p \in Z$

So $x = \frac{\pi}{2} + \pi k = \frac{\pi}{2} + 2\pi p,\ p \in Z$

Answer: $\frac{\pi}{2} + 2\pi p,\ p \in Z$