Conditions

2. (a) when it rains on the surface z= (1/x) + (1/y) +xy at what point(s) will a puddle(s) of water form?
(b) use a computer to produce a graph that supports your analysis in part (a)

Solution

These points will be those, where function is concave down.

Let's find Gesse's Matrix:

$$H(z) = \begin{pmatrix} \frac{\partial^2 z}{dx^2} & \frac{\partial^2 z}{\partial x \partial y} \\ \frac{\partial^2 z}{\partial y \partial x} & \frac{\partial^2 z}{\partial y^2} \end{pmatrix} = \begin{pmatrix} \frac{2}{x^3} & 1 \\ 1 & \frac{2}{y^3} \end{pmatrix}$$

Also

$$grad(z) = (-\frac{1}{x^2} + y, -\frac{1}{y^2} + x)$$

$$\left(-\frac{1}{x^2} + y, -\frac{1}{y^2} + x\right) = (0,0)$$

$$\begin{cases} -\frac{1}{x^2} + y = 0 \\ -\frac{1}{y^2} + x = 0 \end{cases}$$

$$y = \frac{1}{x^2}$$

$$x - x^4 = 0$$

$$x = 1$$

$$y = 1$$

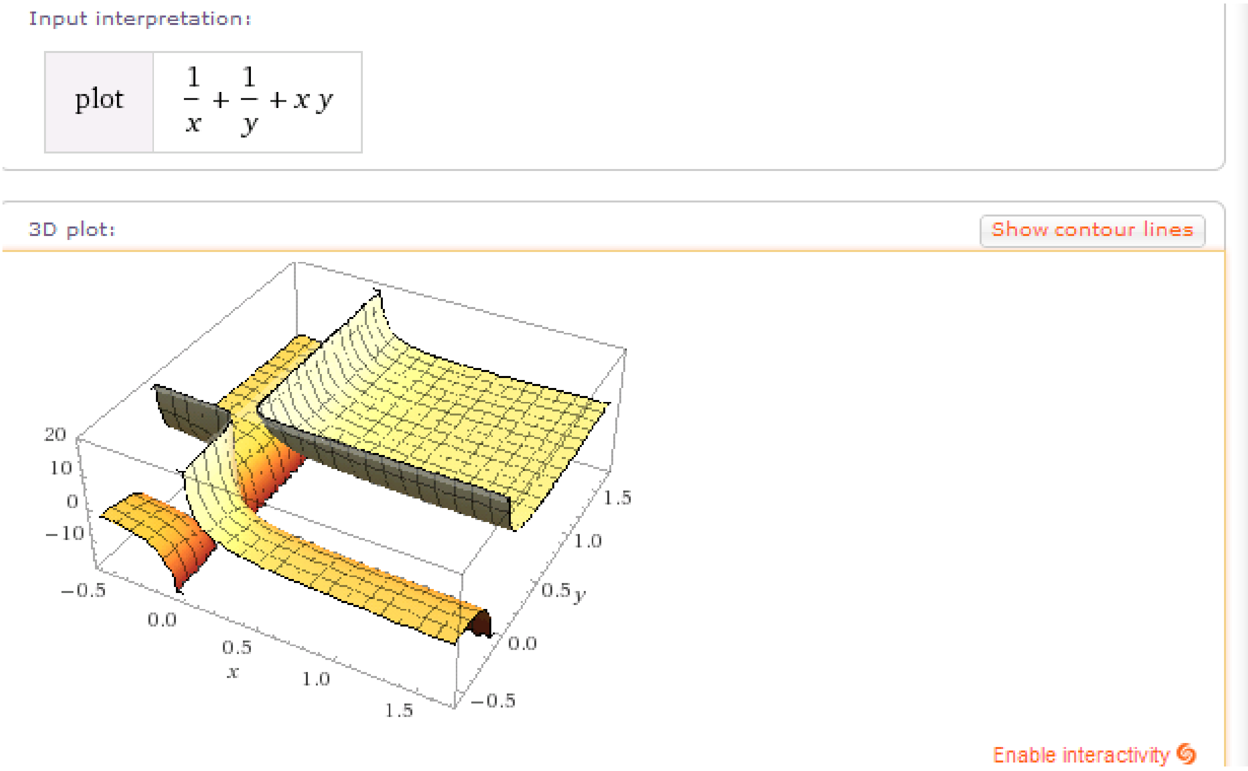
$$H(z(1,1)) = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$$

This matrix is positive definite.

Point (1,1) is a point of local minimum of z(x,y).

The function will be concave down when H(z) is positive definite. So when x>0 and y>0.

Let’s graph a plot:



Indeed, we’ve got that our function is concave down when x>0, y>0.

This supports our previous logic.