

$\sin x + \tan x/2 = 0$.find general solution

Solution.

Substituting $\tan x = \frac{\sin x}{\cos x}$ and factorizing the left part we have

$$\sin x + \frac{\sin x}{2\cos x} = 0 \quad \text{or} \quad \frac{\sin x(2\cos x + 1)}{2\cos x} = 0.$$

From here

$$\cos x \neq 0 \quad \text{and} \quad \sin x = 0 \quad \text{or} \quad 2\cos x + 1 = 0.$$

1) $\sin x = 0 \Rightarrow x = \pi k, k \in \mathbb{Z}.$

2) $2\cos x + 1 = 0 \Rightarrow \cos x = -\frac{1}{2} \Rightarrow x = \pm \arccos\left(-\frac{1}{2}\right) + 2\pi k \Rightarrow x = \pm\left(\pi - \arccos\frac{1}{2}\right) + 2\pi k \Rightarrow$

$$x = \pm \frac{2\pi}{3} + 2\pi k, k \in \mathbb{Z}.$$

Answer: $x = \pi k, x = \pm \frac{2\pi}{3} + 2\pi k, k \in \mathbb{Z}.$