1. Find the equation of the plane that passes through the line of intersection of the planes 4x − 3y − z − 1 = 0 and 2x + 4y + z − 5 = 0 and passes through A(1, -3, 2).

2. Find the equation of the plane that passes through the line of intersection of the planes 4x − 3y − z − 1 = 0 and 2x + 4y + z − 5 = 0 and parallel to the x− axis.

Solution:

1. At first, we find two points or line of intersection of the given planes.

Considering the system of their equations and resolving it with respect to x and y we have

$$\begin{cases} 4x-3y-z-1=0 \\ 2x+4y+z-5=0 \end{cases} \Rightarrow \begin{cases} 4x-3y=1+z \\ 2x+4y=5-z \end{cases} \Rightarrow x=\frac{19+z}{22}, y=\frac{9-3z}{11}.$$

Suppose $z=-19 \Rightarrow x=0, y=6.$ It is the first point.

Suppose $z=3 \Rightarrow x=1, y=0.$ It is the second point.

So, the plane passes through the points $A(1,-3,2)$, $B(0,6,-19)$ and $C(1,0,3)$.

Using now the corresponding form of the equation of a plane

$$\begin{vmatrix} x-x_A & y-y_A & z-z_A \\ x_B-x_A & y_B-y_A & z_B-z_A \\ x_C-x_A & y_C-y_A & z_C-z_A \end{vmatrix}=0$$

and substituting our data we get the equation of the desired plane

$$\begin{vmatrix} x-1 & y-6 & z+19 \\ -1 & 9 & -22 \\ 0 & 3 & 1 \end{vmatrix}=0.$$

2.Consider the vector $\overrightarrow{BC}=(1,-6,22)$,where the points B and C have been found in the previous task, and the direction vector $\vec{s}=(1,0,0)$ of $x-$axis. And find the normal vector $\vec{n}$ of the plane as $\vec{n}=\vec{s}\times\overrightarrow{BC}$.We'll get

$$\vec{n}=\begin{vmatrix} i & j & k \\ 1 & 0 & 0 \\ 1 & -6 & 22 \end{vmatrix}=0i-22j-6k, \text{ so } \vec{n}=(0,-22,-6).$$

So, the plane passes through the point $B(0,6,-19)$ and has the normal vector $\vec{n}=(0,-22,-6)$. Using then a canonical form of a plane we have

$$0(x-0)-22(y-6)-6(z+19)=0$$

or

$$11y + 3z - 9 = 0.$$

Answer: $11y + 3z - 9 = 0$.