

Task Using the rules of differentiation, find the derivatives of functions.

a) $y = 3\sqrt{x} + 7/x$.

We will use the following formulas:

$$(x^n)' = nx^{n-1}.$$

For instance

$$(\sqrt{x})' = (x^{1/2})' = \frac{1}{2}x^{\frac{1}{2}-1} = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}},$$

and

$$\left(\frac{1}{x}\right)' = (x^{-1})' = -x^{-1-1} = -x^{-2} = -\frac{1}{x^2}.$$

Therefore

$$y' = (3\sqrt{x} + 7/x)' = 3(\sqrt{x})' + 7\left(\frac{1}{x}\right)' = \frac{3}{2\sqrt{x}} - \frac{7}{x^2}$$

b) $y = t \cos(2t)$.

We will use the following formulas:

$$(fg)' = f'g + fg',$$

where $f(t) = t$ and $g(t) = \cos(2t)$, and

$$(f(g(t))' = f'(g(t)) * g'(t),$$

for $f = \cos(t)$ and $g = 2t$. So

$$\begin{aligned} y' &= (t \cos(2t))' = (t)' \cos(2t) + t (\cos(2t))' = 1 \cdot \cos(2t) + t \cdot (-\sin(2t)) \cdot 2 \\ &= \cos(2t) - 2t \sin(2t). \end{aligned}$$

c) $y = \frac{x}{x^2+1}$.

We will use the formula:

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

for $f(t) = t$ and $g(t) = x^2 + 1$. So

$$\begin{aligned} y' &= \left(\frac{x}{x^2+1}\right)' = \frac{x' \cdot (x^2+1) - x \cdot (x^2+1)'}{(x^2+1)^2} = \frac{1 \cdot (x^2+1) - x \cdot 2x}{(x^2+1)^2} = \frac{x^2+1-2x^2}{(x^2+1)^2} \\ &= \frac{1-x^2}{(x^2+1)^2} \end{aligned}$$