$\cos ^{-1}$ is a function such that $\cos ^{-1}(t)$ is an angle x from $[0 ; 2 \pi)$ such that $\cos (x)=t$.

We know that $\cos (\pi / 3)=1 / 2$. Also from trigonometry $\cos (\pi-y)=-\cos (y)$ for every real $y$.

Then
$\cos (\pi / 3)=1 / 2=-\cos (\pi-\pi / 3)=-\cos (2 \pi / 3)$
$\cos (2 \pi / 3)=-1 / 2$
Now we get for angle $2 \pi / 3$ equality $\cos (2 \pi / 3)=-1 / 2$. Then by definition $\cos ^{-1}(-1 / 2)=2 \pi / 3$

