Conditions

The digits of Phil's bill have accidentally been reversed. He has been overcharged by an amount that uses the same digits as the correct price. If he spent a whole number of dollars less than $1000, what is the correct price?

Solution. The correct price cannot be a single digit number since Phil would not have been overcharged. If the correct price is a two digit number $10a+b$ then $10b+a-(10a+b)=10a+b$ so $8b=19a$ which is impossible if a and b are non-zero single digits. Thus the correct price is a three digit number $100a+10b+c$ where $c>a$ (so that the difference is non-zero.) Hence $100c+10b+a-(100a+10b+c)=100x+10y+z$ and

$$z=a+10-c, \quad y=b+10-(b+1)=9, \quad x=c-(a+1).$$

Therefore one of a, b or c is 9. Note that $z\neq a$ since $c\neq 10$. Thus if $b=9$ then $a+10-c=c$ and $c-(a+1)=a$ but then $a=8/3$ and $c=19/3$ are not integers. Hence $b\neq 9$ and $c>a$ so $a\neq 9$ hence $c=9$. And $z=b$ so $a+1=b$ and $a=8-a$ so $a=4$ and $b=5$. The correct price is $459.