Question 1. *In the quadrilateral $ABCD$:*

$$CA \perp DB, \quad BC = 6cm, \quad CD = 3cm, DA = 5cm.$$

Find AB.

Solution. Let O be the intersection of CA and DB. By Pythagorean theorem for $\triangle AOB$, $\triangle BOC$, $\triangle COD$, $\triangle DOA$ we have

$$\begin{aligned} OA^2 + OB^2 &= AB^2, \\ OB^2 + OC^2 &= BC^2, \\ OC^2 + OD^2 &= CD^2, \\ OA^2 + OD^2 &= AD^2. \end{aligned}$$

Adding the first 3 equalities we get

$$OA^2 + OD^2 + 2(OB^2 + OC^2) = AB^2 + BC^2 + CD^2.$$

But $OA^2 + OD^2 = AD^2$ and $OB^2 + OC^2 = BC^2$, so

$$AD^2 + 2BC^2 = AB^2 + BC^2 + CD^2,$$

whence

$$AB^2 = AD^2 + BC^2 - CD^2 = 25 + 36 - 9 = 52.$$

Answer: $AB = \sqrt{52} = 2\sqrt{13}$ cm.

□