

**Question 1.** Determine  $|A \cup B \cup C|$  when  $|A| = 50$ ,  $|B| = 500$  and  $|C| = 5000$ , if

(a)  $A \subseteq B \subseteq C$ ;

(b)  $A \cap B = A \cap C = B \cap C = \emptyset$ ;

(c)  $|A \cap B| = |A \cap C| = |B \cap C| = 3$  and  $|A \cap B \cap C| = 1$ .

*Solution.* (a) If  $A \subseteq B \subseteq C$ , then  $A \cup B \cup C = C$ , so  $|A \cup B \cup C| = |C| = 5000$ .

(b) If  $A \cap B = A \cap C = B \cap C = \emptyset$ , then  $|A \cup B \cup C| = |A| + |B| + |C| = 50 + 500 + 5000 = 5550$ .

(c) Now if  $|A \cap B| = |A \cap C| = |B \cap C| = 3$  and  $|A \cap B \cap C| = 1$ , then by inclusion-exclusion formula

$$\begin{aligned} |A \cup B \cup C| &= |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C| \\ &= 50 + 500 + 5000 - 3 - 3 - 3 + 1 \\ &= 5542. \end{aligned}$$

*Answer:*

(a) 5000;

(b) 5550;

(c) 5542.

□