1)Let V= R3. Show that W is a subspace of V where W = {(a, b, 0): a, b ϵ R}, i.e. W is the xy plane consisting of those vectors whose third component is 0.

2)Let V= R3. Show that W is not a subspace of V where W = {(a, b, c): a, b, c ϵ Q}, i.e. W consists of those vectors whose components are rational numbers.

3)Determine whether the vectors v1 = (2, -1, 3), v2 = (4, 1, 2) and v3 = (8, -1, 8) span R3.

4)Use system of linear equations form and row echelon form to show that the vectors (2, -1, 4), (3, 6, 2) and (2, 10, -4) are linearly independent.

Solution.

1) *W* is a subspace of R^3.

For proving this fact it is necessary to show that for any $v_1 = (x1, y1, 0)$ and $v_2 = (x2, y2, 0)$ belonging to *W* and any $a1, a2 \in R$ their linear combination $a1 * v_1 + a2 * v_2 \in W$. We have

$a1*(x1, y1, 0) + a2*(x2, y2, 0) = (a1*x1 + a2*x2, a1*y1 + a2*y2, 0) \in W$. Q.E.D.

2)The set $W = \{(a,b,c,) : a,b,c, \in Q\}$ is not a subspace over the set real numbers as ,for example,

$$\sqrt{2}(a,b,c) = (\sqrt{2}a, \sqrt{2}b, \sqrt{2}c) \notin W ,$$

as $\sqrt{2}a, \sqrt{2}b, \sqrt{2}c \notin Q$.

3)Vectors v_1, v_2, v_3 form a spanning set of R^3 if any vector $v \in R^3$ may be represented as a linear combination of v_1, v_2, v_3.This can be done always if a determinant formed by the vectors v_1, v_2, v_3 is not equal to zero. So, consider

$$\det = \begin{vmatrix} 2 & 4 & 8 \\ -1 & 1 & -1 \\ 3 & 2 & 8 \end{vmatrix}$$

and using Leibniz formula ,for example,we have det=16-12-16-24+4+32=0. The given vectors do not form a spanning set of R^3.

4) Consider a system

$$x(2,-1,4) + y(3,6,2) + z(2,10,-4) = 0$$

or

$$\begin{aligned} 2x + 3y + 2z &= 0 \\ -x + 6y + 10z &= 0 . \\ 4x + 2y - 4z &= 0 \end{aligned}$$

It has only trivial solution if its determinant equals zero. So, forming and computing determinant we have

$$\det = \begin{vmatrix} 2 & 3 & 2 \\ -1 & 6 & 10 \\ 4 & 2 & -4 \end{vmatrix} = -48 + 120 - 4 - 48 - 40 - 12 = -32 \neq 0 .$$

Hence the given vectors are linear independent.

Do the same using row echelon form. With the help of Gaussian elimination we get

$$\left|\begin{array}{ccc|c} 2 & 3 & 2 & 0 \\ -1 & 6 & 10 & 0 \\ 4 & 2 & -4 & 0 \end{array}\right| \to \left|\begin{array}{ccc|c} -1 & 6 & 10 & 0 \\ 2 & 3 & 2 & 0 \\ 4 & 2 & -4 & 0 \end{array}\right| \to \left|\begin{array}{ccc|c} -1 & 6 & 10 & 0 \\ 0 & 15 & 22 & 0 \\ 0 & 26 & 36 & 0 \end{array}\right| \to \left|\begin{array}{ccc|c} -1 & 6 & 10 & 0 \\ 0 & 15 & 22 & 0 \\ 0 & 0 & -32 & 0 \end{array}\right| \Rightarrow x = y = z = 0$$

and vectors are linear independent.