Find set of all possible values of a in [-pi,pi]such that sqrt((1-sina)/(1+sina)) ls equal to (seca – tana)

Solution.

1.Rewrite the equation in the form

$$\sqrt{\frac{1-\sin a}{1+\sin a}}=\frac{1-\sin a}{\cos a} \qquad (1)$$

2.Describe the set of allowed values of *a*.

$$\frac{1-\sin a}{1+\sin a}\geq 0 \ , \qquad (2)$$

$$1+\sin a\neq 0, \qquad (3)$$

$$\frac{1-\sin a}{\cos a}\geq 0, \qquad (4)$$

$$\cos a\neq 0 \ . \qquad (5)$$

3.Solve the equation (1).

Squaring both parts

$$\frac{1-\sin a}{1+\sin a}=\frac{(1-\sin a)^2}{(\cos a)^2},$$

taking out common multiple

$$(1-\sin a)\left(\frac{1-\sin a}{\cos^2 a}-\frac{1}{1+\sin a}\right)=0,$$

and reducing to the same denominator

$$(1-\sin a)\left(\frac{1-\sin^2 a-\cos^2 a}{\cos^2 a(1+\sin a)}\right)=0$$

we have

$$(1-\sin a)*0=0.$$

So, the values of *a* satisfying requirements (2)-(5) are solutions of the equation (1).

4.Solve (2)-(5).

We have

$$\sin a\neq -1\Rightarrow a\neq -\frac{\pi}{2}+2\pi k, k\in Z,$$

$$\cos a\neq 0\Rightarrow a\neq \pm\frac{\pi}{2}+2\pi k \ ,$$

$$\cos a>0\Rightarrow a\in\left(-\frac{\pi}{2}+2\pi k,\frac{\pi}{2}+2\pi k\right).$$

So,

$$a\in\left(-\frac{\pi}{2}+2\pi k,\frac{\pi}{2}+2\pi k\right).$$

5.Finally taking in mind that $a\in[-\pi,\pi]$ we get $k=0$ and $a\in(-\frac{\pi}{2},\frac{\pi}{2})$.

Answer: $a\in(-\frac{\pi}{2},\frac{\pi}{2})$.