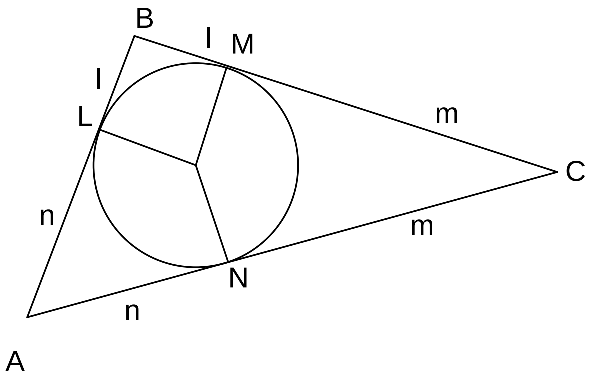

Question:

If l,m,n are the distances of vertices of a triangle from the corresponding points of contact with the incircle, then prove that lmn/(l+m+n)=r^2
(r is inradius)

Solution:

For proof using next formula for radius incircle:

$$r = \sqrt{\frac{(p-a)(p-b)(p-c)}{p}} \quad (1)$$

where

$a = AB = n + l,$

$b = BC = l + m,$

$c = AC = n + m,$

$p = \frac{1}{2}(a + b + c) = \frac{1}{2}(n + l + l + m + n + m) = l + n + m.$

Insert a, b, c, p into (1):

$$r = \sqrt{\frac{(l+n+m-(n+l))(l+n+m-(l+m))(l+n+m-(n+m))}{l+n+m}}$$

$$r = \sqrt{\frac{m \cdot n \cdot l}{l+n+m}}$$

$$r^2 = \frac{mnl}{l+n+m}$$