

**Task:**

A square and a rectangle have the same perimeter of 80 cm. If the difference between their areas is 100 cm<sup>2</sup>, find the sides of the rectangle.

**Solution:**

The perimeter of a square whose four sides have length  $s$  is given by the formula:

$$P = 4s$$

And the area is:

$$A = s^2$$

We know that  $P_s = 80$  cm, so we can find the side length of the square:

$$s = \frac{P_s}{4} = \frac{80}{4} = 20 \text{ (cm)}$$

The area of the square is:

$$A_s = 20^2 = 400 \text{ (cm}^2\text{)}$$

Proceeding from this we can find the area of the rectangle:

$$A_s - A_r = 100$$

$$A_r = A_s - 100$$

$$A_r = 400 - 100$$

$$A_r = 300 \text{ (cm}^2\text{)}$$

Perimeter of a rectangular is given by the formula:

$P_r = 2a + 2b$ , where  $a$  – length and  $b$  – width.

And the area is:  $A_r = a \cdot b$

So we get the system of equations:  $\begin{cases} 2a + 2b = 80 \\ a \cdot b = 300 \end{cases}$

Solve it:

$$\begin{cases} 2a + 2b = 80 \\ a \cdot b = 300 \end{cases} \begin{cases} 2b = 80 - 2a \\ a \cdot b = 300 \end{cases} \begin{cases} b = 40 - a \\ a \cdot (40 - a) = 300 \end{cases} \begin{cases} b = 40 - a \\ 40a - a^2 = 300 \end{cases} \begin{cases} b = 40 - a \\ a^2 - 40a + 300 = 0 \end{cases}$$

$$\begin{cases} b = 40 - a \\ a^2 - 40a + 300 = 0 \end{cases} \begin{cases} b_1 = 40 - 10 \\ a_1 = 10 \end{cases} \begin{cases} b_1 = 30 \\ a_1 = 10 \end{cases} \begin{cases} b_2 = 40 - 30 \\ a_2 = 30 \end{cases} \begin{cases} b_2 = 10 \\ a_2 = 30 \end{cases}$$

**Answer:** 30 cm and 10 cm.