

Task:

A square and a rectangle have the same perimeter of 80 cm. If the difference between their area is 100 cm square, find the sides of rectangle

Solution:

The perimeter of a square whose four sides have length s is given by the formula:

$$P = 4s$$

And the area is:

$$A = s^2$$

We know that $P_s = 80$ cm, so we can find sides length of the square:

$$s = \frac{P_s}{4} = \frac{80}{4} = 20 \text{ (cm)}$$

The area of the square is:

$$A_s = 20^2 = 400 \text{ (cm}^2\text{)}$$

Proceeding from this we can find the area of the rectangle:

$$A_s - A_r = 100$$

$$A_r = A_s - 100$$

$$A_r = 400 - 100$$

$$A_r = 300 \text{ (cm}^2\text{)}$$

Perimeter of a rectangular is given by the formula:

$$P_r = 2a + 2b, \text{ where } a - \text{length and } b - \text{width.}$$

And the area is: $A_r = a \cdot b$

So we get the system of equations:
$$\begin{cases} 2a + 2b = 80 \\ a \cdot b = 300 \end{cases}$$

Solve it:

$$\begin{cases} 2a + 2b = 80 \\ a \cdot b = 300 \end{cases} \begin{cases} 2b = 80 - 2a \\ a \cdot b = 300 \end{cases} \begin{cases} b = 40 - a \\ a \cdot (40 - a) = 300 \end{cases} \begin{cases} b = 40 - a \\ 40a - a^2 = 300 \end{cases} \begin{cases} b = 40 - a \\ a^2 - 40a - 300 = 0 \end{cases}$$

$$\begin{cases} b = 40 - a \\ a^2 - 40a - 300 = 0 \end{cases} \begin{cases} b_1 = 40 - 10 \\ a_1 = 10 \end{cases} \begin{cases} b_1 = 30 \\ a_1 = 10 \end{cases} \begin{cases} b_2 = 40 - 30 \\ a_2 = 30 \end{cases} \begin{cases} b_2 = 10 \\ a_2 = 30 \end{cases}$$

Answer: 30 cm and 10 cm.