The total surface area S of a right circular cylinder is related to the base radius r and height h by the equation $S = 2\pi r^2 + 2\pi rh$

a) How is dS/dt related to dr/dt and dh/dt if neither r nor h is a constant?

b) How is dS/dt related to dr/dt if h is constant?

c) How is dS/dt related to dr/dt if r is constant?

Solution:

a)

$$\frac{dS}{dt} = \frac{d(2\pi r^2)}{dr} \cdot \frac{dr}{dt} + \frac{d(2\pi r)}{dr} \cdot \frac{dr}{dt} \cdot h + \frac{d(2\pi r)}{dh} \cdot \frac{dh}{dt} \cdot r =$$

$$= 4\pi r \cdot \frac{dr}{dt} + 2\pi h \cdot \frac{dr}{dt} + 2\pi r \cdot \frac{dh}{dt} = 2\pi(2r + h) \cdot \frac{dr}{dt} + 2\pi r \cdot \frac{dh}{dt}$$

So

$$\frac{dS}{dt} = 2\pi(2r + h) \cdot \frac{dr}{dt} + 2\pi r \cdot \frac{dh}{dt}$$

b) If h is constant then

$$\frac{dh}{dt} = 0.$$

Thus

$$\frac{dS}{dt} = 2\pi(2r + h) \cdot \frac{dr}{dt}$$

c) If r is constant then

$$\frac{dr}{dt} = 0.$$

Thus

$$\frac{dS}{dt} = 2\pi r \cdot \frac{dh}{dt}$$