Conditions

The total surface area S of a right circular cylinder is related to the base radius r and height h by the equation S=2πr^2+2πrh

a) How is dS/dt related to dr/dt and dh/dt if neither r nor h is a constant?

b) How is dS/dt related to dr/dt if h is constant?

c) How is dS/dt related to dr/dt if r is constant?

Solution

a)

we must take the derivative by t, looking at r and h as functions of t:

$$r = r(t); h = h(t)$$

$$S = 2\pi r^2 + 2\pi rh$$

$$\frac{dS}{dt} = \mathbf{4\pi r} \cdot \frac{\boldsymbol{dr}}{\boldsymbol{dt}} + \mathbf{2\pi r}\frac{\boldsymbol{dh}}{\boldsymbol{dt}} + \mathbf{2\pi h}\frac{\boldsymbol{dr}}{\boldsymbol{dt}}$$

b) if h is a constant, then:

$$\frac{dS}{dt} = 4\pi r \cdot \frac{dr}{dt} + 0 + 2\pi h\frac{dr}{dt} = \mathbf{2\pi(2r + h)}\frac{\boldsymbol{dr}}{\boldsymbol{dt}}$$

c) if r is a constant, then:

$$\frac{dS}{dt} = 0 + 2\pi r\frac{dh}{dt} + 0 = \mathbf{2\pi r}\frac{\boldsymbol{dh}}{\boldsymbol{dt}}$$