Conditions

Prove that if both set A&B are bounded above then
i) AUB is also bounded above and
ii) sup (AUB) = max { sup(A), sup(B) }

Can anyone help me solve this problem? i understand what is the meaning of bounded above and have little understanding what sup means. Thanks

Solution

If we are saying about sets bounded **above**, then we have A and B being one-dimensional sets.

Let's write down, what does it mean:

$\exists X: \forall a \epsilon A \; a \leq X$

$\exists Y: \forall b \epsilon B \; b \leq X$

Now let's write down, what does mean AUB:

$AUB = \{c: c\epsilon A \; or \; c\epsilon B\}$

If so, then $\forall c\epsilon AUB \;\; c \leq \max\{a, b\}$ for some fixed a and b. And this means that AUB is bounded above.

If set is bounded above, then it has a supremum.

Supremum means the smallest element from a set of all elements, which have a property like X and Y (see above)

So,

$\sup(A) \leq X \; \forall X: a \leq X \; \forall a\epsilon A$

$\sup(B) \leq Y \; \forall Y: b \leq Y \; \forall b\epsilon B$

$\sup(AUB) \leq Z: \forall Z: c \leq Z \; \forall z\epsilon AUB$

As $c \leq \max\{a, b\}$ for all a,b with properties shown above, then the smallest element from a set of all these maximums is exactly the maximum between two supremums of A and B.

Q.E.D.