

When you stretch your arms horizontally, the tip of your fingers are 160cm away from each other. If they are 170cm from the tip of your toes, and 85cm from the top of your head, how tall are you? Give the answer to the nearest tenth.

Solution.

Let us denote the tips of the fingers as A and C, the tip of the toe as D and the top of the head as B. Connecting the dots A,B,C,D we will get the quadrilateral in which diagonals AC and BD are perpendicular. In problem it is necessary to find the growth of human, namely, the length of diagonal BD. The basic data are: AC=160 cm, AD=CD=170 cm, AB=BC=85 cm.

On the one hand the area of the quadrilateral ABCD equals

$$A = \frac{1}{2} AC * BD.$$

From this

$$BD = \frac{2A}{AC} = \frac{2A}{160} = \frac{A}{80}. \quad (1)$$

On the other hand the area of ABCD equals the sum of the areas of the triangles ABC and ADC. For its finding we use Heron's formula

$$A = \sqrt{p(p-a)(p-b)(p-c)},$$

where a,b,c are the lengths of the sides and p is half the perimeter.

So, for the triangle ABC we have

$$p = \frac{85+85+160}{2} = 165; A_1 = \sqrt{165(165-160)(165-85)(165-85)} = \sqrt{165*5*80*80} = 400\sqrt{33},$$

for the triangle ACD,-

$$p = \frac{160+170+170}{2} = 250; A_2 = \sqrt{250(250-160)(250-170)(250-170)} = \sqrt{250*90*80*80} = 400*30.$$

Then the area of the quadrilateral ABCD equals

$$A = A_1 + A_2 = 400\sqrt{33} + 400*30 = 400(\sqrt{33} + 30)$$

and according to (1), the length of BD equals

$$BD = \frac{400(\sqrt{33} + 30)}{80} = 5(\sqrt{33} + 30) \approx 5(30 + 5.74) = 178.7(\text{cm}).$$

Answer: 178.7(cm).