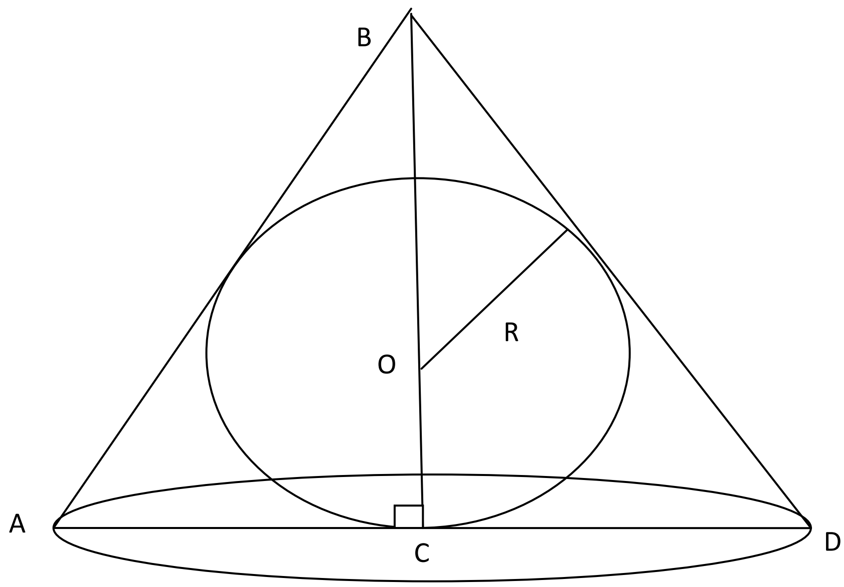
You are given a cone of height of 1 unit. The base angle is 40 degrees. Determine what radius (of a perfect sphere) should have to maximize the volume within the cone. The volume above is irrelevant.

Solution:



We have

$$BC = 1\ (unit)$$

$$\angle BAC = 40°$$

We must find R and

$$V = \frac{4}{3}\pi R^3$$

We know next formula

$$R = \frac{2S}{P}$$

where S - area of ΔABD, P - perimeter of one. So

$$S = \frac{1}{2}BC \cdot AD = BC \cdot AC = BC \cdot BC \cdot ctg\angle BAC = ctg(40°)(square\ units)$$

$$P = AD + BD + BA = 2AC + 2BA = 2\left(BC \cdot ctg\angle BAC + \frac{BC}{sin\angle BAC}\right) =$$
$$= \frac{2(1 + cos(40°))}{sin(40°)}\ (units)$$

Then

$$R = \frac{2ctg(40°)}{\frac{2(1 + cos(40°))}{sin(40°)}} = \frac{cos(40°)}{1 + cos(40°)}\ (units) \approx 0.4338(units)$$

So we have

$$V = \frac{4}{3}\pi\left(\frac{cos(40°)}{1 + cos(40°)}\right)^3 \approx 0.3419(cubic\ units)$$

Answer:

$$R = \frac{cos(40°)}{1 + cos(40°)}\ (units) \approx 0.4338(units)$$

$$V = \frac{4}{3}\pi\left(\frac{cos(40°)}{1 + cos(40°)}\right)^3 \approx 0.3419(cubic\ units)$$