**Question 1.** *The operation $*$ is defined on the set $\mathbb{Q}$ of rational numbers by $a * b = ab + a + b$, $a, b \in \mathbb{Q}$. Find the element of $\mathbb{Q}$ which does not have an inverse under $*$.*

*Solution.* First of all note that $*$ is commutative and 0 is the identity under $*$. Indeed,

$$a * 0 = 0 * a = a \cdot 0 + a + 0 = a.$$

Now take $a \in \mathbb{Q}$ and suppose there is $b \in \mathbb{Q}$ such that

$$a * b = 0 \Leftrightarrow ab + a + b = 0 \Leftrightarrow b(a - 1) = -a.$$

We see that if $a = 1$, then this equality becomes $0 = -1$, which is a contradiction. So, $a = 1$ is not invertible. But if $a \neq 1$, then $b = \frac{a}{1-a} \in \mathbb{Q}$ is the inverse of $a$.
*Answer:* 1. $\square$