

the diagram shows part of the curve $y=2-18/(2x+3)$, which crosses the x-axis at A and the y-axis at B. The normal to the curve at A crosses the y-axis at C. The normal to the curve at A crosses the y-axis at C. Find the length of BC

Solution.

1. Find the coordinates of the point A.

Supposing $y=0$ from the equation of the curve we have

$$0=2-18/(2x+3) \Rightarrow 2=18/(2x+3) \Rightarrow 2(2x+3)=18 \Rightarrow x=3.$$

So, A(3,0).

2. Find the coordinates of the point B.

Supposing $x=0$ from the equation of the curve we have

$$y=2-18/3=-4.$$

So, B(0,-4).

3. Find the slope of the curve at the point A.

It is equal to the derivative of $y(x)$ at this point. So,

$$y'(x)=36/(2x+6)^2$$

and $y'(3)=4/9$.

4. Through the point A draw a straight line perpendicular to the given curve.

An equation of this line is

$$y=-1/y'(3)*(x-3)$$

or

$$y=-9/4*x+27/4. \quad (1)$$

5. Find the coordinates of the point C.

The point C is the point of intersection of the line (1) and y-axis. So, supposing in (1) $x=0$ we have $y=27/4$. So, the point C(0,27/4).

6. Find the length of BC.

As x-coordinates of the points B and C are equal the length of BC equals

$$BC=|y_B - y_C| = |-4 - 27/4| = 43/4.$$

Answer: $BC=27/4$.