

Conditions

$$\int \frac{1}{x - \sqrt{x+2}} dx$$

Solution

$$\int \frac{1}{x - \sqrt{x+2}} dx$$

$$\text{Substitute: } u = \sqrt{x+2}; du = \frac{1}{2\sqrt{x+2}}$$

$$\int \frac{1}{x - \sqrt{x+2}} dx = 2 \int \frac{1}{u^2 - u - 2} du = \int \frac{1}{u^2 - u - 2} du + \int \frac{2u - 1}{u^2 - u - 2} du$$

$$\int \frac{2u - 1}{u^2 - u - 2} du = \ln(u^2 - u - 2) + c$$

$$\int \frac{1}{u^2 - u - 2} du = \int \frac{1}{\left(u - \frac{1}{2}\right)^2 - \frac{9}{4}} du = \frac{2}{3} \tanh^{-1}\left(\frac{1}{3} - \frac{2u}{3}\right) + c$$

$$\int \frac{1}{x - \sqrt{x+2}} dx = \frac{4}{3} \ln(x - \sqrt{x+2}) + \frac{2}{3} \ln(1 + \sqrt{x+2}) + c$$

Answer:

$$\frac{2}{3} (2 \ln(x - \sqrt{x+2}) + \ln(1 + \sqrt{x+2})) + c$$