Theorem 1 (Fermat's little theorem)**.** *If p is a prime number, then for any positive integer n we have $n^p \equiv n \mod p$.*

Question 1. *Let n be an integer divisible by 9. Prove that $n^7 \equiv n \mod 63$.*

Solution. We need to prove that $n^7 - n$ is divisible by 63. Since $n^7 - n = n(n^6 - 1)$ and n is divisible by 9, then $n^7 - n$ is divisible by 9. As 7 and 9 are relatively prime, it is sufficient to prove that $n^7 - n$ is divisible by 7. But this immediately follows from Fermat's little theorem, because 7 is a prime number. □