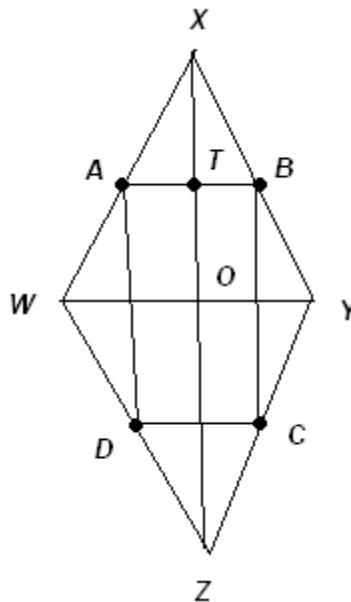


## Conditions

How do you Prove that the midpoints of the sides of a kite form a rectangle?

## Solution

Consider a graph:



As we know, a kite has a form of two isosceles triangles with a common base.

As  $WX=XY$ , and A and B are midpoints, then AB is parallel to WY. XO is a height, so angle XTB is 90 degrees. The angle XOY is 90 degrees too, as the diagonals of rhombus are intersecting with the right angle.

Consider the triangle XOY:

$$\angle XYO = 90^\circ; \angle OYX = 90^\circ - \angle OXY$$

$$\angle OYX = \angle ABX, \text{ because } AB \text{ and } WY \text{ are parallel.}$$

As B and C are midpoints, then XZ is parallel to BC, then:

$$\angle OYB = \angle OXY$$

The angle

$$\begin{aligned}\angle XBY &= 180^\circ = \angle XBA + \angle ABC + \angle OBY = \angle OYX + \angle ABC + \angle OXY \\ &= \angle ABC + 90^\circ - \angle OXY + \angle OXY = \angle ABC + 90^\circ\end{aligned}$$

$$180^\circ = \angle ABC + 90^\circ$$

$$\angle ABC = 90^\circ$$

Analogically we can consider other 3 triangle and prove that each angle of ABCD is  $90^\circ$ . And this is a rectangle by the definition of rectangle.

Q.E.D.