form a PDE by elimination f and g from z= f(x2-y)+g(x2+y)

Solution

Given = $f(x^2 - y) + g(x^2 + y)$ (1)

Differentiating (1) partially with respect to x,

$\frac{\partial z}{\partial x} = f'(u) * 2x + g'(v) * 2x = 2x\left(f'(u) + g'(v)\right)$ (2)

Where $u = x^2 - y$ and $v = x^2 + y$

Differentiating (1) partially with respect to y,

$\frac{\partial z}{\partial y} = f'(u) * (-1) + g'(v) * 1 = g'(v) - f'(u)$ (3)

Differentiating (2) partially with respect to x,

$\frac{\partial^2 z}{\partial x^2} = \left(2x\left(f'(u) + g'(v)\right)\right)' = 2\left(f'(u) + g'(v)\right) + (2x)^2\left(f''(u) + g''(v)\right)$ (4)

Differentiating (3) partially with respect to y,

$\frac{\partial^2 z}{\partial y^2} = \left(g'(v) - f'(u)\right)' = \left(f''(u) + g''(v)\right)$ (5)

From (2), (4) and (5) we have

$$\frac{\partial^2 z}{\partial x^2} = (2x)^2 \frac{\partial^2 z}{\partial y^2} + \frac{1}{x}\frac{\partial z}{\partial x}$$

Answer: $\boldsymbol{\frac{\partial^2 z}{\partial x^2} = (2x)^2 \frac{\partial^2 z}{\partial y^2} + \frac{1}{x}\frac{\partial z}{\partial x}}$**.**