

Evaluate $\int \frac{2-5x-x^2}{x^2-x-6} dx$

$$\int \frac{2-5x-x^2}{x^2-x-6} dx = -\int \frac{x^2-x-6+6x+4}{x^2-x-6} dx = -\int dx - \int \frac{6x+4}{(x+2)(x-3)} dx$$

$$\frac{6x+4}{(x+2)(x-3)} = \frac{A}{x+2} + \frac{B}{x-3}$$

$$Ax - 3A + Bx + 2B = 6x + 4$$

$$\begin{cases} A + B = 6 \\ -3A + 2B = 4 \end{cases}$$

$$A = \frac{8}{5}$$

$$B = \frac{22}{5}$$

$$\int \frac{2-5x-x^2}{x^2-x-6} dx = -x - \frac{8}{5} \int \frac{dx}{x+2} - \frac{22}{5} \int \frac{dx}{x-3} = -x - \frac{8}{5} \ln(x+2) - \frac{22}{5} \ln(x-3) + C$$