du/dt of (t^-.5)e^(((-x^2)/((4k^2)t))
Also d2u/dx2

Solution

$$u(x,t)=\frac{1}{\sqrt{\mathrm{t}}}\mathrm{e}^{\left(\frac{(-\mathrm{x}^2)}{(4\mathrm{k}^2)\mathrm{t}}\right)}$$

1. $\frac{du}{dt}=\frac{d}{dt}\left(\frac{1}{\sqrt{\mathrm{t}}}\mathrm{e}^{\left(\frac{(-\mathrm{x}^2)}{(4\mathrm{k}^2)\mathrm{t}}\right)}\right)=\frac{d}{dt}\left(\frac{1}{\sqrt{\mathrm{t}}}\right)\mathrm{e}^{\left(\frac{(-\mathrm{x}^2)}{(4\mathrm{k}^2)\mathrm{t}}\right)}+\frac{1}{\sqrt{\mathrm{t}}}\frac{d}{dt}\left(\mathrm{e}^{\left(\frac{(-\mathrm{x}^2)}{(4\mathrm{k}^2)\mathrm{t}}\right)}\right)$

a. $\frac{d}{dt}\left(\frac{1}{\sqrt{\mathrm{t}}}\right)=\frac{d}{dt}\left(t^{-\frac{1}{2}}\right)=-\frac{1}{2}\ t^{-\frac{3}{2}}=-\frac{1}{2t\sqrt{t}}$

b. $\frac{d}{dt}\left(\mathrm{e}^{\left(\frac{(-\mathrm{x}^2)}{(4\mathrm{k}^2)\mathrm{t}}\right)}\right)=\mathrm{e}^{\left(\frac{(-\mathrm{x}^2)}{(4\mathrm{k}^2)\mathrm{t}}\right)}*\frac{d}{dt}\left(\frac{(-\mathrm{x}^2)}{(4\mathrm{k}^2)\mathrm{t}}\right)=\frac{(-\mathrm{x}^2)}{(4\mathrm{k}^2)}\mathrm{e}^{\left(\frac{(-\mathrm{x}^2)}{(4\mathrm{k}^2)\mathrm{t}}\right)}\frac{d}{dt}\left(\frac{1}{t}\right)=\frac{(-\mathrm{x}^2)}{(4\mathrm{k}^2)}\mathrm{e}^{\left(\frac{(-\mathrm{x}^2)}{(4\mathrm{k}^2)\mathrm{t}}\right)}\frac{d}{dt}\left(t^{-1}\right)=$
$-\frac{1}{t^2}\frac{(-\mathrm{x}^2)}{(4\mathrm{k}^2)}\mathrm{e}^{\left(\frac{(-\mathrm{x}^2)}{(4\mathrm{k}^2)\mathrm{t}}\right)}$

So we have

$$\frac{du}{dt}=-\frac{1}{2t\sqrt{t}}\mathrm{e}^{\left(\frac{(-\mathrm{x}^2)}{(4\mathrm{k}^2)\mathrm{t}}\right)}+\frac{1}{\sqrt{\mathrm{t}}}\left(-\frac{1}{t^2}\frac{(-\mathrm{x}^2)}{(4\mathrm{k}^2)}\mathrm{e}^{\left(\frac{(-\mathrm{x}^2)}{(4\mathrm{k}^2)\mathrm{t}}\right)}\right)=\frac{\mathrm{e}^{\left(\frac{(-\mathrm{x}^2)}{(4\mathrm{k}^2)\mathrm{t}}\right)}}{2t\sqrt{t}}\left(\frac{(\mathrm{x}^2)}{(2\mathrm{k}^2)\mathrm{t}}-1\right)$$

2. $\frac{d^2u}{dx^2}=\frac{d}{dx}\left(\frac{du}{dx}\right);\ \frac{du}{dx}=\frac{d}{dx}\left(\frac{1}{\sqrt{\mathrm{t}}}\mathrm{e}^{\left(\frac{(-\mathrm{x}^2)}{(4\mathrm{k}^2)\mathrm{t}}\right)}\right)=\frac{1}{\sqrt{\mathrm{t}}}\mathrm{e}^{\left(\frac{(-\mathrm{x}^2)}{(4\mathrm{k}^2)\mathrm{t}}\right)}\frac{d}{dx}\left(\frac{(-\mathrm{x}^2)}{(4\mathrm{k}^2)\mathrm{t}}\right)=\frac{1}{\sqrt{\mathrm{t}}}\mathrm{e}^{\left(\frac{(-\mathrm{x}^2)}{(4\mathrm{k}^2)\mathrm{t}}\right)}\frac{(-2x)}{4k^2t}$

Then

$$\frac{d^2u}{dx^2}=\frac{d}{dx}\left(\frac{1}{\sqrt{\mathrm{t}}}\mathrm{e}^{\left(\frac{(-\mathrm{x}^2)}{(4\mathrm{k}^2)\mathrm{t}}\right)}\frac{(-2x)}{4k^2t}\right)=-\frac{1}{2k^2t\sqrt{\mathrm{t}}}\frac{d}{dx}\left(x\mathrm{e}^{\left(\frac{(-\mathrm{x}^2)}{(4\mathrm{k}^2)\mathrm{t}}\right)}\right)$$

$$\frac{d}{dx}\left(x\mathrm{e}^{\left(\frac{(-\mathrm{x}^2)}{(4\mathrm{k}^2)\mathrm{t}}\right)}\right)=x\frac{d}{dx}\left(\mathrm{e}^{\left(\frac{(-\mathrm{x}^2)}{(4\mathrm{k}^2)\mathrm{t}}\right)}\right)+\mathrm{e}^{\left(\frac{(-\mathrm{x}^2)}{(4\mathrm{k}^2)\mathrm{t}}\right)}\frac{dx}{dx}=\mathrm{e}^{\left(\frac{(-\mathrm{x}^2)}{(4\mathrm{k}^2)\mathrm{t}}\right)}+x\mathrm{e}^{\left(\frac{(-\mathrm{x}^2)}{(4\mathrm{k}^2)\mathrm{t}}\right)}\frac{(-2x)}{4k^2t}=$$

$$=\mathrm{e}^{\left(\frac{(-\mathrm{x}^2)}{(4\mathrm{k}^2)\mathrm{t}}\right)}\left(1-\frac{(x^2\ )}{2k^2t}\right)$$

So

$$\frac{d^2u}{dx^2}=\frac{\mathrm{e}^{\left(\frac{(-\mathrm{x}^2)}{(4\mathrm{k}^2)\mathrm{t}}\right)}}{2k^2t\sqrt{\mathrm{t}}}\left(\frac{(\mathrm{x}^2)}{(2\mathrm{k}^2)\mathrm{t}}-1\right)$$

Answer: 1. $\boldsymbol{\frac{\mathrm{e}^{\left(\frac{(-\mathrm{x}^2)}{(4\mathrm{k}^2)\mathrm{t}}\right)}}{2t\sqrt{t}}\left(\frac{(\mathrm{x}^2)}{(2\mathrm{k}^2)\mathrm{t}}-1\right)}$**; 2.** $\boldsymbol{\frac{\mathrm{e}^{\left(\frac{(-\mathrm{x}^2)}{(4\mathrm{k}^2)\mathrm{t}}\right)}}{2k^2t\sqrt{\mathrm{t}}}\left(\frac{(\mathrm{x}^2)}{(2\mathrm{k}^2)\mathrm{t}}-1\right)}$.