

Factor $a^3 + b^2 + c^2$ over $a^2 + b + c$ no line between them, no signs of operation.

Explanation

Factoring is the opposite process of multiplying polynomials, in order to return to a unique string of polynomials of lesser degree whose product is the original polynomial, is the simplest way to solve equations of higher degree. When we factor a polynomial, we are looking for simpler polynomials that can be multiplied together to give us the polynomial that we started with. If all of the terms in a polynomial contain one or more identical factors, combine those similar factors into one monomial, called the greatest common factor, and rewrite the polynomial in factored form.

In our case, $a^3 + b^2 + c^2$ we can divide on $a^2 + b + c$

$$\begin{array}{r}
 \underline{a^3 + b^2 + c^2} \quad | \quad a^2 + b + c \\
 \underline{a^3 + ab + ac} \quad | \quad a + b + c \\
 -ab - ac + b^2 \\
 \hline
 \underline{a^2b + bc + b^2} \\
 -ab - ac - a^2b - bc + c^2 \\
 \hline
 \underline{a^2c + bc + c^2} \\
 -ab - ac - a^2b - bc - a^2c - bc
 \end{array}$$

Thus we can write, $(a^2 + b + c)(a + b + c) + (-ab - ac - a^2b - bc - a^2c) =$

$$= (a^2 + b + c)(a + b + c) + (-a^2b - a^2c - 2bc - ab - ac)$$

$$\text{Answer: } (a^2 + b + c)(a + b + c) + (-a^2b - a^2c - 2bc - ab - ac)$$

Check:

$$\begin{aligned}
 (a^2 + b + c)(a + b + c) + (-ab - ac - a^2b - bc - a^2c - bc) &= (a^3 + a^2b + a^2c + ab + \\
 &+ b^2 + bc + ac + bc + c^2 - ab - ac - a^2b - bc - a^2c - bc) = (a^3 + a^2b + c^2)
 \end{aligned}$$

If we consider the second polynomial $a^2 + b + c$, we can provide a solution in the following way - can be expressed as the sum of the squares:

$$a^2 + b + c = (a + (\sqrt{b + c})) (a + (\sqrt{b + c})) - 2a\sqrt{b + c}$$

Also we can check:

$$\begin{aligned}
 (a + (\sqrt{b + c})) (a + (\sqrt{b + c})) - 2a\sqrt{b + c} &= (a^2 + a\sqrt{b + c} + a\sqrt{b + c} + \sqrt{b + c} \times \\
 &\sqrt{b + c} - 2a\sqrt{b + c}) = (a^2 + 2a\sqrt{b + c} + b + c - 2a\sqrt{b + c}) = (a^2 + b + c)
 \end{aligned}$$

Answer:

$$a^2 + b + c = (a + (\sqrt{b + c})) (a + (\sqrt{b + c})) - 2a\sqrt{b + c}$$