

Condition of the problem: A circle of radius R is inscribed in an equilateral triangle, all of whose sides are of length S. A mathematician claims that the length of the line segment joining the center of the circle to any of the 3 triangle vertices is equal in length to the diameter of the circle. Your first task is to confirm that claim if it is true. Otherwise, you need to prove that the claim is not true.

Then you are to perform the appropriate calculations to determine:

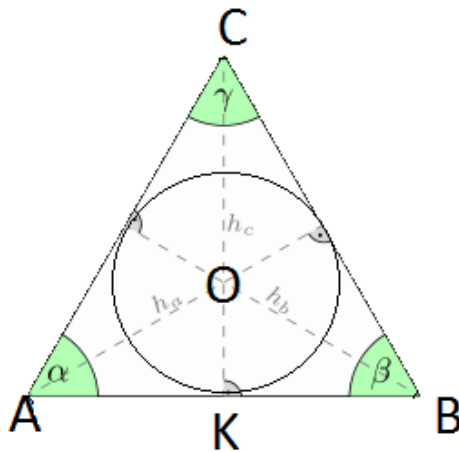
The area of the circle in terms of S

The area of the triangle in terms of S

The area of the triangle in terms of R.

Note: If a appears in your work and/or in your solutions, just leave it that way. Do not convert to 1.732. Also, be sure to rationalize all denominators. And, of course, leave answers in terms of π when π appears.

Solution:



In this triangle $OK=R$,

See on triangle OBK . Angle $OBK=30^\circ$, angle $OKB = 90^\circ$, and angle $KOC=60^\circ$.

$$OB = \frac{OK}{\sin(30^\circ)} = \frac{R}{\frac{1}{2}} = 2 * R = D, \quad D - \text{diameter}$$

So, mathematician was right.

$$AK = \frac{S}{2}, \quad OK = AK * \tan(30^\circ) = \frac{S\sqrt{3}}{6}, \quad \text{so}$$

$$\text{area of circle} = \frac{\pi S^2}{12}$$

$$\text{area of triangle} = \frac{1}{2} * AB * CK = \frac{1}{2} * S * \frac{S\sqrt{3}}{2} = \frac{S^2\sqrt{3}}{4}$$

$$AK = OK * \tan(60^\circ) = R\sqrt{3}, \quad \text{so } AB = 2R\sqrt{3}, CK = 3R, \quad \text{and area of triangle} = 3R^2\sqrt{3}$$