Condition of the problem: Find the average value of the function over the given interval and every (all) value(s) of in the interval for which the function equals its average value.

$$f(x) = \frac{4x^2 + 1}{x^2}$$

over the interval [1,3].

Solution: by definition average of function $f(x)$ over interval $[a, b]$ equals to

$$\bar{f} = \frac{1}{b-a}\int_a^b f(x)dx$$

For our function we get

$$\bar{f} = \frac{1}{3-1}\int_1^3 \frac{4x^2+1}{x^2}dx = \frac{1}{2}\int_1^3 \left(4 + \frac{1}{x^2}\right)dx = \frac{1}{2}(4x - \frac{1}{x})\Big|_1^3 = \frac{1}{2}\left(12 - \frac{1}{3} - 4 + 1\right) = \frac{13}{3}$$

Now we can find value(s) of in the interval for which the function equals its average value by solving equation $f(x) = \bar{f},\ x \in [1,3]$

$$\frac{4x^2+1}{x^2} = \frac{13}{3}$$

$$4x^2 + 1 = \frac{13}{3}x^2$$

$$\frac{1}{3}x^2 = 1$$

$$x = \sqrt{3}$$

Answer: $\bar{f} = \frac{13}{3}, x = \sqrt{3}$