

Answer on question # 2564 – Math – Statistics and Probability

The time between shots at goal in a football (soccer) game, X , has an exponential distribution with a mean of 9 shots every twenty minutes. Determine the following

- a) What is the probability of the time between shots being greater than 5 minutes?
- b) What is the probability of more than 20 shots in 40 minutes?
- c) In 6 non-overlapping 5 minute intervals, what is the probability of at least one 5 minute period with no shots at goal?

Answer:

X is the time between shots; and density function is

$$f(x) = \begin{cases} 0, & x < 0 \\ \lambda e^{-\lambda x}, & x \geq 0 \end{cases}$$

Where $\lambda = \frac{1}{Ex}$. From the task we get the mean $Ex = \frac{20}{9}$, therefore $\lambda = \frac{9}{20}$ and

$$f(x) = \begin{cases} 0, & x < 0 \\ \frac{9}{20} e^{-\frac{9}{20}x}, & x \geq 0 \end{cases}$$

Then we get

a) $P(x > 5) = 1 - P(x \leq 5) = 1 - \int_0^5 \frac{9}{20} e^{-\frac{9}{20}x} dx = \left(1 + e^{-\frac{9}{20}x}\right) \Big|_0^5 \approx 0.105;$

b) $P(x < 2) = \int_0^2 \frac{9}{20} e^{-\frac{9}{20}x} dx = \left(e^{-\frac{9}{20}x}\right) \Big|_0^2 \approx 0.59.$

The probability that this event will occur 20 times in a row is $(0.59)^{20} \approx 0.000029;$

- c) This probability equals 1 minus the probability of event that all intervals between shots less than 5 six times.

$$1 - (P(x < 5))^6 = 1 - \left(\int_0^5 \frac{9}{20} e^{-\frac{9}{20}x} dx\right)^6 = 1 - \left(\left(e^{-\frac{9}{20}x}\right) \Big|_0^5\right)^6 \approx 0.4874.$$

Answer: a) 0.105; b) 0.000029; c) 0.4874.