

$$1. \frac{(\sec x - \tan x)^2 + 1}{\csc x (\sec x - \tan x)} = 2 \tan x,$$

$$\sec x = \frac{1}{\cos x}, \csc x = \frac{1}{\sin x}, \tan x = \frac{\sin x}{\cos x}, \cos^2 x + \sin^2 x = 1$$

$$\begin{aligned} \frac{(\sec x - \tan x)^2 + 1}{\csc x (\sec x - \tan x)} &= \frac{\left(\frac{1}{\cos x} - \frac{\sin x}{\cos x}\right)^2 + 1}{\frac{1}{\sin x} \left(\frac{1}{\cos x} - \frac{\sin x}{\cos x}\right)} = \frac{\frac{1}{\cos^2 x} ((1 - \sin x)^2 + \cos^2 x)}{\frac{1}{\sin x} \left(\frac{1}{\cos x} - \frac{\sin x}{\cos x}\right)} = \\ &= \frac{\frac{\sin x}{\cos^2 x} ((1 - \sin x)^2 + \cos^2 x)}{\frac{1}{\cos x} (1 - \sin x)} = \frac{\frac{\sin x \cos x}{\cos^2 x} ((1 - \sin x)^2 + \cos^2 x)}{(1 - \sin x)} = \\ &= \frac{\frac{\sin x}{\cos x} ((1 - \sin x)^2 + \cos^2 x)}{(1 - \sin x)} = \frac{\frac{\sin x}{\cos x} (1 - 2\sin x + \sin^2 x + \cos^2 x)}{(1 - \sin x)} = \\ &= \frac{\frac{\sin x}{\cos x} (1 - 2\sin x + 1)}{(1 - \sin x)} = \frac{\frac{2\sin x}{\cos x} (1 - \sin x)}{(1 - \sin x)} = \frac{2\sin x}{\cos x} = 2 \tan x. \end{aligned}$$

$$2. \frac{1 + \cos x + \sin x}{1 + \cos x - \sin x} = \sec x + \tan x$$

$$\begin{aligned} \frac{1 + \cos x + \sin x}{1 + \cos x - \sin x} &= \frac{1 + \cos x + \sin x}{1 + \cos x - \sin x} \times \frac{1 + \cos x - \sin x}{1 + \cos x - \sin x} = \\ &= \frac{1 + \cos x - \sin x + \cos x + \cos^2 x - \sin x \cos x + \sin x + \sin x \cos x - \sin^2 x}{1 + \cos x - \sin x + \cos x + \cos^2 x - \sin x \cos x - \sin x - \sin x \cos x + \sin^2 x} = \\ &= \frac{1 + 2\cos x + \cos^2 x - \sin^2 x}{1 + 2\cos x - 2\sin x - 2\sin x \cos x + 1} = \frac{\cos^2 x + \sin^2 x + 2\cos x + \cos^2 x - \sin^2 x}{2 + 2\cos x - 2\sin x - 2\sin x \cos x} = \\ &= \frac{2\cos^2 x + 2\cos x}{2((1 + \cos x) - \sin x(1 + \cos x))} = \\ &= \frac{2\cos x(\cos x + 1)}{2(1 + \cos x)(1 - \sin x)} = \frac{\cos x}{1 - \sin x} \times \frac{1 + \sin x}{1 + \sin x} = \frac{\cos x(1 + \sin x)}{1 - \sin^2 x} = \frac{\cos x(1 + \sin x)}{\cos^2 x} = \\ &= \frac{1 + \sin x}{\cos x} = \frac{1}{\cos x} + \frac{\sin x}{\cos x} = \sec x + \tan x. \end{aligned}$$