

find q if $x = (\sec q + \tan q)$ and $y = b(\sec q - \tan q)$

Solution

Multiply x by y :

$$xy = b(\sec q + \tan q)(\sec q - \tan q) = b(\sec^2 q - \tan^2 q) = b\left(\frac{1}{\cos^2 q} - \tan^2 q\right)$$
$$= \left|\frac{1}{\cos^2 q} = 1 + \tan^2 q\right| = b * 1 = b \rightarrow b = xy$$

$$x = (\sec q + \tan q) = \frac{1}{\cos q} + \frac{\sin q}{\cos q} = \frac{1 + \sin q}{\cos q}$$

Let's use $\begin{cases} \cos\left(\frac{\pi}{2} - q\right) = \sin q \\ \sin\left(\frac{\pi}{2} - q\right) = \cos q \end{cases}$. Then

$$x = \frac{1 + \sin q}{\cos q} = \frac{1 + \cos\left(\frac{\pi}{2} - q\right)}{\sin\left(\frac{\pi}{2} - q\right)} = \cot\frac{\left(\frac{\pi}{2} - q\right)}{2}, \text{ because of } \cot\frac{q}{2} = \frac{1 + \cos q}{\sin q}$$

So

$$x = \cot\left(\frac{\pi}{4} - \frac{q}{2}\right) \rightarrow \left(\frac{\pi}{4} - \frac{q}{2}\right) = \cot^{-1} x \rightarrow q = 2\left(\frac{\pi}{4} - \cot^{-1} x\right) = \frac{\pi}{2} - 2 \cot^{-1} x.$$

Answer: $\frac{\pi}{2} - 2 \cot^{-1} x$.