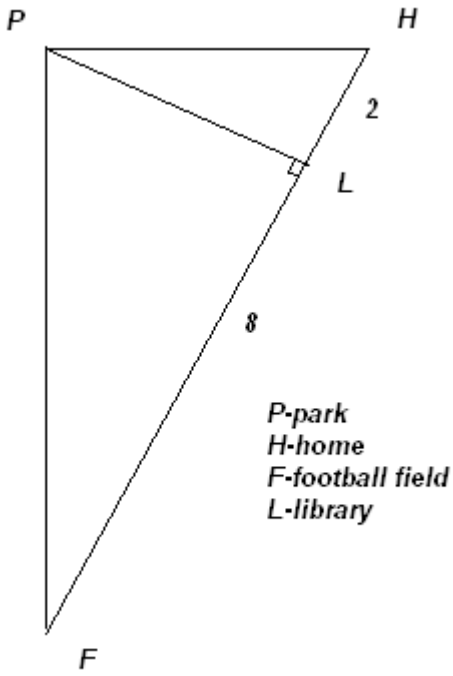
Conditions

Kristen lives directly east of the park. The football field is directly south of the park. The library sits on the line formed between Kristen's home and the football field at the exact point where an altitude to the right triangle formed by her home, the park, and the football field could be drawn. The library is 2 miles from her home. The football field is 8 miles from the library. How far is library from the park?

How far is the park from the football field?

Solution

Consider a graph:



Using the Pythagorean Theorem, from the triangle HPF we have:

$$PF^2 + PH^2 = 100$$

Using the Pythagorean Theorem, from the triangle PHL we have:

$$PL^2 = PH^2 - HL^2 = PH^2 - 4$$

Using the Pythagorean Theorem, from the triangle PLF we have:

$$PL^2 = PF^2 - FL^2 = PF^2 - 64$$

So,

$$PH^2 - 4 = PF^2 - 64$$

$$PH^2 - PF^2 = -60$$

We can construct the system of 2 equations with 2 variables:

$$\begin{cases} PF^2 + PH^2 = 100 \\ PH^2 - PF^2 = -60 \end{cases}$$

$$PF^2 - 60 + PF^2 = 100$$

$$2PF^2 = 160$$

$$PF^2 = 80; PF = 4\sqrt{5}$$

$$PH^2 = 100 - 80 = 20; PH = 2\sqrt{5}$$

$$PL^2 = PH^2 - HL^2 = PH^2 - 4 = 20 - 4 = 16; PL = 4$$

So, we have found, that the distance between a library and a park is 4 miles and the distance from a football field to a park is $4\sqrt{5}$