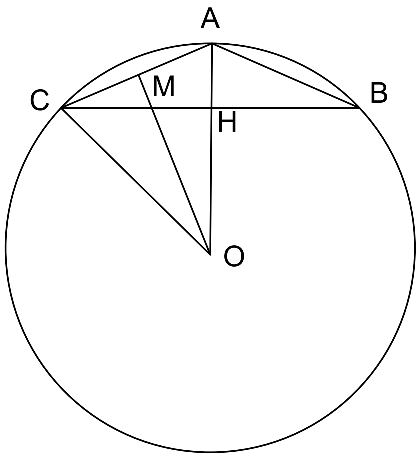
If an isosceles triangle ABC, AB=AC=6cm is inscribed in a circle of radius 9cm.

Find the area of the triangle.



Solution:

$S = \frac{1}{2} BC \times AH = HB \times AH$

Theorem: The angle subtended at the centre of a circle by an arc is twice any angle at the circumference standing on the same arc.

So $\angle AOC = 2\angle ABC$

ΔAOC is an isosceles triangle (OA and OC are radiuses)

So if OM is a perpendicular to AC, then AM=MC and $\angle AOM = \angle MOC = \frac{1}{2}\angle AOC = \angle ABC$

ΔAOM and ΔABH are similar triangle ($\angle AOM = \angle ABC$, $\angle AMO$ and $\angle AHB$ are right –AA)

So $\frac{AM}{AO} = \frac{AH}{AB}$

$AM = \frac{1}{2}AC = 3\ cm \quad AO = R = 9\ cm \quad AB = 6\ cm$

$$AH = \frac{AM \times AB}{AO} = \frac{3 \times 6}{9} = 2\ cm$$

Using Pythagoras' theorem

$HB = \sqrt{AB^2 - AH^2} = \sqrt{36 - 4} = 4\sqrt{2}\ cm$

$S = 4\sqrt{2} \times 2 = 8\sqrt{2}\ cm^2$

Answer: the area of the triangle is $8\sqrt{2}\ cm^2$, the area of the circle is $S_c = \pi R^2 = 81\pi\ cm^2$