Question #25414

$$\tan x = \frac{2}{3} \Rightarrow x = \pi n + \tan^{-1}\left(\frac{2}{3}\right), \qquad n \in \mathbb{Z}$$

$$\sec x = -\frac{\sqrt{13}}{3} \sim \frac{1}{\cos x} = -\frac{\sqrt{13}}{3} \sim \cos x = -\frac{3}{\sqrt{13}} \Rightarrow x = 2\pi n \pm \cos^{-1}\left(-\frac{3}{\sqrt{13}}\right), \qquad n \in \mathbb{Z}$$

$$1 + \tan^2 x = \frac{1}{\cos^2 x};\ 1 + \tan^2 x = \sec^2 x$$

Substitute with original data:

$$1 + \left(\frac{2}{3}\right)^2 = \left(-\frac{\sqrt{13}}{3}\right)^2;\ 1 + \frac{4}{9} = \frac{13}{9};\ \frac{13}{9} = \frac{13}{9}$$

We get a true equality for all x. But we need to check signs (because sec(x) was with "-"). For answer of system equastions we can take both asnwers of original equastions. Also look for preservation period and sign.

$$\begin{cases} \tan x = \frac{2}{3} \\ \sec x = -\frac{\sqrt{13}}{3} \end{cases} \sim \begin{cases} x = \pi n + \tan^{-1}\left(\frac{2}{3}\right) \\ x = 2\pi n \pm \cos^{-1}\left(-\frac{3}{\sqrt{13}}\right) \end{cases} \Rightarrow x = 2\pi n - \pi + \tan^{-1}\left(\frac{2}{3}\right), \qquad n \in \mathbb{Z}$$