

Conditions

Integral of $1/(x^{1/2} + x^{1/4}) dx$

Solution

Here we can use substitution $u = \sqrt[4]{x}$:

$$\int \frac{1}{\sqrt{x} + \sqrt[4]{x}} dx = \left[u = \sqrt[4]{x} \quad du = \frac{1}{4\sqrt[4]{x^3}} dx \right] = 4 \int \frac{u^3}{u^2 + u} dx = 4 \int \frac{u^2}{u + 1} dx =$$

Now do a long division: $\frac{u^2}{u+1} = u - 1 + \frac{1}{u+1}$

$$= 4 \int \left(u - 1 + \frac{1}{u + 1} \right) dx = 2u^2 - 4u + 4 \ln(u + 1) + c = 2\sqrt{x} - 4\sqrt[4]{x} + 4 \ln(\sqrt[4]{x} + 1) + c$$