

Question:

$x = \sec t + \tan t$, show that $x + 1/x = 2 \sec t$ (t is an angle)

Solution:

$$x = \sec(t) + \tan(t) = \frac{1}{\cos(t)} + \frac{\sin(t)}{\cos(t)} = \frac{1 + \sin(t)}{\cos(t)}$$

$$\begin{aligned}x + \frac{1}{x} &= \frac{x^2 + 1}{x} = \frac{\left(\frac{1 + \sin(t)}{\cos(t)}\right)^2 + 1}{\frac{1 + \sin(t)}{\cos(t)}} = \frac{\frac{(1 + \sin(t))^2}{\cos^2(t)} + 1}{\frac{1 + \sin(t)}{\cos(t)}} = \frac{(1 + \sin(t))^2 + \cos^2(t)}{\cos(t)(1 + \sin(t))} \\&= \frac{1 + 2\sin(t) + \sin^2(t) + \cos^2(t)}{\cos(t)(1 + \sin(t))} = \frac{1 + 2\sin(t) + 1}{\cos(t)(1 + \sin(t))} = \frac{2(1 + \sin(t))}{\cos(t)(1 + \sin(t))} \\&= \frac{2}{\cos(t)}\end{aligned}$$