Conditions

x =sec t + tan t, show that x + 1/x = 2 sec t

Solution

$x = \sec t + \tan t$

Consider:

$$x + \frac{1}{x} = \sec t + \tan t + \frac{1}{\sec t + \tan t} =$$

$$\sec t = \frac{1}{\cos t}; \tan t = \frac{\sin t}{\cos t}$$

$$= \frac{1}{\cos t} + \frac{\sin t}{\cos t} + \frac{1}{\frac{1}{\cos t} + \frac{\sin t}{\cos t}} = \frac{\sin t + 1}{\cos t} + \frac{\cos t}{1 + \sin t} = \frac{(\sin t + 1)^2 + \cos^2 t}{\cos t\,(1 + \sin t)} =$$

$$= \frac{\mathbf{\sin^2 t} + 2\sin t + 1 + \mathbf{\cos^2 t}}{\cos t\,(1 + \sin t)} = \frac{2\sin t + 2}{\cos t\,(1 + \sin t)} = 2\frac{1 + \sin t}{\cos t\,(1 + \sin t)} = \frac{2}{\cos t} = \mathbf{2\,sec t}$$